A Study of Lateral Boundary Conditions for The NRL's Coupled Ocean/Atmosphere Mesoscale Prediction System (COAMPS)

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LONG-TERM GOALS

Our long-term goal is to improve lateral boundary conditions in local area models.

OBJECTIVES

The treatment of lateral boundaries in regional models has been a perennial problem since the early days of numerical weather prediction. In a limited-area model the lateral edges are not physical boundaries of the flow but constitute artificial constraints imposed by computational considerations. Hence they do not have a physical counterpart. We must impose conditions at these artificial boundaries in order to solve the problem in an efficient and accurate manner.
We have developed high order non-reflecting boundary conditions for the dispersive Klein-Gordon equation and the linearized shallow water equations.

APPROACH

The treatment of lateral boundaries in regional models has been a perennial problem since the early days of numerical weather prediction. In a limited-area model, the lateral edges are not physical boundaries of the flow but constitute artificial constraints imposed by computational considerations. Hence they do not have a physical counterpart. We must impose conditions at these artificial boundaries in order to solve the problem in an efficient and accurate manner.
Several good review papers were written on the topic of both actual and artificial boundary conditions. A general discussion was presented by Turkel [24], while a review of non-reflecting boundary conditions presenting the issue in a uniform manner is that of Givoli [5]. Non-reflecting boundary

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conditions aim to achieve the following goals for a problem defined in a computational domain Ω whose artificial boundary is Γ:

- The problem in Ω together with the boundary condition on Γ should be mathematically well posed and be a good approximation of the original problem.
- The boundary condition on Γ should be highly compatible with the numerical discretization scheme used in the interior computational domain Ω.

It is also important that the numerical method used together with the boundary condition should result in a numerically stable scheme, while the amount of spurious reflection generated by the boundary condition on Γ should be small. Another consideration is that the implementation of the boundary condition should not lead to a large computational effort.

Earlier on, Elvius and Sundstrom [2] derived discrete equations based on an extrapolation formula linking neighboring points for the nonlinear shallow-water equations and proceeded to analyze the stability of the resulting finite difference scheme for the mixed initial boundary value problem.

Recently a comprehensive and extensive review of artificial boundary conditions was presented by Tsynkov [23]. He shows that any algorithm for getting artificial boundary conditions can be viewed as a compromise between locality and practical efficacy but implying insufficient accuracy. On the other hand highly accurate nonlocal boundary techniques often result in impractical algorithms.

Gustafsson and Sundstrom [6] treated the linearized shallow-water equations using a Fourier transform in all but one Cartesian direction and used energy estimates to treat the resulting one-dimensional systems to generate well-posed boundary conditions for the half-space problem.

Tsynkov [23] also shows that the method of introducing sponge layers near the computational boundaries occupies an intermediate position between local and nonlocal approaches. This is because on the one hand, there are no global integral relations along the boundary. On the other hand a certain amount of nonlocality persists since the computational domain is artificially magnified and when numerical simulations are conducted the model equations inside the sponge layer are solved using a numerical method close or identical to the one employed inside the computational domain.

Two basic approaches have been used for dealing with limited-area boundary conditions. Simple boundary conditions for hyperbolic equations consist in having a specification requiring that the characteristics leaving from the interior domain be zero. In addition one must combine the absorbing boundary conditions with the given in flow conditions. A strictly mathematical treatment of absorbing boundary conditions (ABC) for hyperbolic equations was presented by Engquist and Majda [3,4] based on pseudo-differential operators subsequently expanded to non-local operators to get approximate localized well-posed ABC 's.

These methods can be viewed as a generalization of the Sommerfeld radiation condition and the characteristic approach. A second alternative approach is the use of sponge or damping layers to damp out disturbances prior to their reaching the artificial boundary.

A variation of this process is to construct a layer where the outgoing waves will slow down rather than decay. Hence, the waves will not reflect back into the limited-area forecast domain of interest except at very late times (see Perkey and Kreitzberg [22], Davies [1], Israeli and Orszag [20]).

The COAMPS system involves an atmosphere and an ocean model (see Hodur [19]).

The equations governing each of these models are solved in a finite computational domain.

Thus, there is a need to apply appropriate boundary conditions on the remote boundaries.

Lateral boundary conditions implemented in COAMPS today are, in order of complexity,

1. Fixed conditions,
2. Periodic conditions,
3. Zero-order (Sommerfeld-like) radiation conditions, and

4. The Davies Lateral Sponge Layers [1].
While this remote-boundary treatment of Davies [1] is efficient and may be sufficiently accurate in some cases of interest, it is not so robust in that it is not directly associated with the notion of convergence. More precisely, the following observations can be made:

1. When using either of the above schemes it is hard to estimate and control the error. The numerical solution may be very accurate with respect to the discrete model; however the discrete model itself may be a rough approximation of the 'true 'model in the large domain due to the artificial truncation of the large domain and the use of the approximate remote boundary condition.

2. If one uses the COAMPS model to solve a given problem and then wishes to improve the accuracy of the solution, it is not sufficient to refine the mesh -this just takes care of the discretization error -but at the same time it is necessary to enlarge the computational domain. This is hard to do in practice, because it necessitates the construction of a new set of grids (containing a large number of grid points), and the calculation of the system of equations for the new grids from the start.

3. The schemes used today require some parameter "tuning "(e.g. the damping parameters and width of the Davies layer). There is no universal tuning in this context, so a certain set of parameters that works well for one problem may not yield satisfactory results for another problem.

WORK COMPLETED

Consider the shallow water equations (SWEs) in a semi-infinite channel. For simplicity we assume that the channel has a flat bottom and that there is no advection, although these assumptions will be removed in later stages of this proposed study. We do take into account rotation (Coriolis) effects. A Cartesian coordinate system (*x, y*) is introduced such that the channel is parallel to the *x* direction. The SWEs are (see [21]):

$$\frac{\partial \mathrm{u}}{\partial \mathrm{t}} + \mu u \frac{\partial u}{\partial x} + \mu v \frac{\partial u}{\partial y} - f v = - g \frac{\partial h}{\partial x} \qquad (1)$$

$$\frac{\partial v}{\partial t} + \mu u \frac{\partial v}{\partial x} + \mu v \frac{\partial v}{\partial y} + f u = - \mathrm{g} \frac{\partial h}{\partial y} \qquad (2)$$

$$\frac{\partial h}{\partial t} + \mu u \frac{\partial h}{\partial x} + \mu v \frac{\partial h}{\partial y} + (\mathrm{h}_0 + \mu h) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0. \qquad (3)$$

Here t is time, $u(x, y, t)$ and $v(x, y, t)$ are the unknown velocities in the x and y directions, h_0 is the given water layer thickness (in the direction normal to the xy plane), $h(x, y, t)$ is the unknown water elevation above h_0, f is the Coriolis parameter, and g is the gravity acceleration. The parameter μ is 1 for the nonlinear SWEs, and is 0 for the linearized SWEs with vanishing mean flow. This form doesn't include the linearized shallow water equations with non zero mean flow. Such a case, as well as the stratified medium, was discussed in our publication. We shall consider the latter as a special case in the sequel. Boundary conditions on the north and south channel walls are v=0, on the west wall we prescribe h and as $x \to \infty$ the solution is known to be bounded and not to include any incoming waves. To complete the problem, initial values for u, v, h are given at $t=0$ in the entire domain. It can be shown that for the linear case, one can rewrite this as the Klein-Gordon equation

$$\frac{\partial^2 h}{\partial t^2} - C_0^2 \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) + f^2 h = 0 \qquad (4)$$

In a series of papers [11]-[15], we have shown how to construct high order non-reflecting boundary conditions (NRBC) on the artificial east boundary.

$$\prod_{j=1}^{J}\left(\frac{\partial}{\partial t} + C_j \frac{\partial}{\partial x}\right) h = 0 \qquad (5)$$

The idea is based on the work of Higdon [18]. The main advantages are as follows:

1. The Higdon NRBCs are very *general*, namely they apply to a variety of wave problems, in one, two and three dimensions and in various configurations.
2. They form a *sequence* of NRBCs of increasing order. This enables one, in principle (leaving implementational issues aside for the moment), to obtain solutions with unlimited accuracy.
3. The Higdon NRBCs can be used, without any difficulty, for *dispersive* wave problems and for problems with layers. Most other available NRBCs are either designed for non-dispersive media (as in acoustics and electromagnetics) or are of low order (as in meteorology and oceanography).
4. For certain choices of the parameters, the Higdon NRBCs are equivalent to NRBCs that are derived from rational approximation of the dispersion relation (the Engquist-Majda conditions being the most well known example). Higdon has proved this in [18] and in earlier papers. Thus, the Higdon NRBCs can be viewed as generalization of rational-approximation NRBCs.

The scheme we developed is different than the original Higdon scheme [18] in the following ways:

1. The discrete Higdon conditions were developed in the literature up to third order only, because of their algebraic complexity, which increases rapidly with the order. We have shown how to easily implement these conditions to an *arbitrarily high order*. The scheme is coded once and for all for any order; the order of the scheme is simply an input parameter.
2. The Higdon NRBCs involve some parameters, which must be chosen. Higdon [18] discusses some general guidelines for their manual a-priori choice by the user. We have shown how these parameters can be chosen *automatically*. They may either be constant, or may change adaptively during the solution process.
3. We have shown how to improve the discretization of the Higdon NRBCs using *higher-order Finite Difference stencils*.
4. We have shown how the Higdon NRBCs may be incorporated in a *Finite Element* scheme.

RESULTS

We have developed high order NRBCs for the Klein-Gordon equation with or without the use of auxiliary variables [7-9]. We have used both finite differences and finite elements [11]. In the latter case, auxiliary variables are required. We have generalized our idea to the linearized SWE with zero mean flow [10,12,13]. We also allowed NRBC on all 4 sides of the lateral boundaries [15]. Later we have added advection and stratification [14,16,17]. In all these linear cases we have shown the benefit of high order NRBCs.

IMPACT/APPLICATIONS

We are working on implementation of second order NRBC into COAMPS.

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